JOURNAL OF PROPULSION AND POWER
Vol. 24, No. 4, July–August 2008

Role of Superconducting Shields in Electrodynamic Propulsion

Juan R. Sanmartín*
Universidad Politécnica de Madrid, 28040 Madrid, Spain
and
Enrico C. Lorenzini†
University of Padova, 35131 Padova, Italy

DOI: 10.2514/1.30433

An electrodynamic tether can propel a spacecraft through a planetary magnetized plasma without using propellant. In the classical embodiment of an electrodynamic tether, the ambient magnetic field exerts a Lorentz force on the current along the tether, the ambient plasma providing circuit closure for the current. A suggested propulsion scheme would hypothetically eliminate tether performance dependence on the plasma density by using a full wire loop to close the current circuit, and a superconductor to shield a loop segment from the external uniform magnetic field and cancel the Lorentz force on that segment. Here, we use basic electromagnetic laws to explain how such a scheme cannot produce a net force. Because there is no net current in the superconducting shield, the circulation of the magnetic field along a closed line outside the full cross section, in its plane, is just due to the current flowing in the loop segment. The presence of the superconducting shield simply moves the Lorentz force from the shielded loop segment to the shield itself and, as a result, the total magnetic force, acting on full loop plus shield, remains zero.

I. Introduction

ELECTRODYNAMIC tether (EDT) applications, including spacecraft deorbiting, thrusting, and power generation, require a planetary magnetized plasma for current exchange and circuit closure, and for both inducing a tether current and exerting a Lorentz force on it [1]. Tether performance thus depends on ambient plasma density n_e and magnetic field B. If either n_e or B is low enough, performance could be unacceptably weak. These limitations appear intrinsic to the EDT concept.

However, modifications of that concept have been suggested, with the purpose of trying to eliminate either the need for a magnetic field, by having the spacecraft produce and carry its own field onboard [2], or the need for ambient current closure, by using a full wire loop to close the circuit [3]. We limit our discussion here to this second scheme. There is, of course, a basic difficulty in the wire-loop scheme: a magnetic field presenting negligible spatial variations in a region with the size of the tether, as in the actual case of interest, would exert no force on the loop [4].

Attaining a net force had been thought possible by diamagnetically shielding a segment of the loop with a material of high magnetic permeability, so that the actual magnetic field inside the wire would be nonuniform along the loop. It was then shown, however, that the forces on the wire segment and shield add to the same force acting on the segment if left unshielded, and so no net force on the loop could be achieved [3]. It has now been suggested, nonetheless, that a superconducting shield might work [5]. Here, we use basic electromagnetic laws to explain how such a scheme cannot produce a net force either.

II. Wire Loop Versus Electrodynamic Tether

The Lorentz force exerted by a uniform field on any finite, steady-current distribution is zero. Using charge conservation, $\nabla \cdot \bar{J} = 0$, the current density $\bar{J}(\bar{r})$ may be written as

$$\bar{J} \equiv \bar{J} \cdot \nabla \bar{r} = \bar{J} \cdot \nabla \bar{r} + (\nabla \cdot \bar{J}) \bar{r} \equiv \nabla \cdot (\bar{J}\, \bar{r}) \tag{1}$$

$\nabla \bar{r}$ being the unit tensor. Integrating Eq. (1) over a volume that includes the entire cloud of current and using Gauss's theorem yields

$$\int \mathrm{d}V \bar{J} = \int \mathrm{d}V \nabla \cdot (\bar{J}\, \bar{r}) = 0 \tag{2}$$

From the Lorentz force per unit volume $\bar{J} \wedge \bar{B}$, the total force on the cloud due to a uniform field $\bar{B}_0$ does indeed vanish:

$$\bar{F} = \int \mathrm{d}V \bar{J} \wedge \bar{B}_0 = \left(\int \mathrm{d}V \bar{J} \right) \wedge \bar{B}_0 = 0 \tag{3}$$

This result applies to a wire loop as just a particular case.

The forces on segments of the loop can, of course, be different from zero and this feature can be used, for example, to stretch a wire loop in orbit [6], the net force vanishing, however. That forces on parts of a current cloud need not be zero is used in the EDT concept, where integrating Eq. (1) over just the tether volume yields

$$\int_{\text{tether}} \mathrm{d}V \bar{J} = \int_{\text{tether}} \mathrm{d}V \nabla \cdot (\bar{J}\, \bar{r}) = \int_{S1,S2} \mathrm{d}\bar{S} \cdot \bar{J}\, \bar{r} = L\bar{I} \tag{4}$$

Here S_1, S_2 are the two end cross sections, with $\bar{I}$ pointing along the tether (assumed straight and insulated for simplicity) in the direction of the current I. In the case of dc current closure through the plasma (but also for the actual case of steady wave-emission closure [7]), one must also have

$$\int_{\text{plasma}} \mathrm{d}V \bar{J} = -L\bar{I} \tag{5}$$

Although the forces on tether and ambient plasma are opposite and equal, a net magnetic power on the tether/plasma system arises from the difference in velocities,

$$(L\bar{I} \wedge \bar{B}_0) \cdot \bar{v}_{\text{tether}} + (-L\bar{I} \wedge \bar{B}_0) \cdot \bar{v}_{\text{plasma}} \equiv -\bar{I} \cdot \bar{E}_m L \tag{6}$$

where the so-called motional electric field

$$\bar{E}_m \equiv (\bar{v}_{\text{tether}} - \bar{v}_{\text{plasma}}) \wedge \bar{B}_0 \tag{7}$$

Received 13 February 2007; revision received 16 February 2008; accepted for publication 3 March 2008.

*Professor, Physics Department/Escuela Técnica Superior de Ingenieros Aeronáuticos, Plaza Cardenal Cisneros 3.

†Professor, Department of Mechanical Engineering, Via Venezia 1.

is the field, in the tether frame, at the highly conductive ambient plasma.

Equation (6) shows that the (Lorentz) mechanical work on tether and plasma, and the work done through the electromotive forces induced as they move, add to zero net work, as required from magnetic forces [8]. In the simplest case of tether parallel to the motional field, there is a potential difference $E_m L$ across the ends of the tether, driving a current in it. If all impedances in the tether circuit are passive, implying $\bar{I} \cdot \bar{E}_m > 0$, the magnetic field will take energy out of the motion of tether and plasma into the tether electrical circuit; the Lorentz force on the tether would be thrust, nonetheless, if $\bar{v}_{\mathrm{tether}}$ is opposite the relative velocity, $\bar{v}_{\mathrm{tether}} - \bar{v}_{\mathrm{plasma}}$. A counter-electromotive force may always be used to invert the direction of the current.

III. Magnetic Force on a Current-Carrying Superconductor

Because Newton's third law holds for the mutual forces between any two clouds of steady current [4], a current-carrying wire loop in a uniform field $\bar{B}_0$ will exert no net force either on the distant current distribution of effectively unlimited extent producing $\bar{B}_0$. In the proposed scheme, that field would supposedly exert a net force on a loop if just modified by having a segment shielded by a superconductor [5]; this would mean that either the loop would now exert a net force on the current distribution producing $\bar{B}_0$ or the third law would fail to hold, breaking momentum conservation for the combined system. In what follows, we will show that none of the aforementioned holds.

The proposed scheme is based on the *facts* that a superconductor is perfectly diamagnetic (no magnetic field inside it) and capable of transporting a current (with zero resistance), the result apparently being that no magnetic force would act on its current. The preceding two statements, however, are overly simplified. The ampere equation, which has general validity for steady conditions,

$$\nabla \wedge \bar{B} = \mu_0 \bar{J} \tag{8}$$

shows that there is a magnetic field wherever there is a current. The fact is, a superconductor may support a field $\bar{B}$ in very thin surface layers (thickness typically on the order of 10^{-5} cm), while any current is carried in such layers. That is, current and field must, and do, coincide spatially in a superconductor too, and result in possibly intense volume forces in thin layers, given as

$$-\mu_0^{-1} \bar{B} \wedge (\nabla \wedge \bar{B}) \tag{9}$$

The pioneering analysis of London was a model of those layers [9].

Loops modified, first by simply making a segment superconductor as considered in this section, and next by enclosing a segment in a superconductor, to be discussed in Sec. IV, do exhibit such thin-layer forces. We consider the simplest geometry for a loop segment, which is a tape of length $L \gg$ width $w \gg$ thickness h, and two cases: 1) there is an external magnetic field B_{a0} parallel to the long sides of the cross section but no current (Fig. 1a); 2) there is a current I_{b0} along the tape but no external magnetic field (Fig. 1b).

In Fig. 1, we place the tape segment cross section in the x–y plane. With $w \gg h$, both $\bar{B}$ and $\bar{J}$ will only vary in the x direction, their components lying along the y and z axes, respectively. Equation (8) then reduces to

$$\frac{\mathrm{d}B_y}{\mathrm{d}x} = \mu_0 J_z \tag{10}$$

Figure 2 shows the simple distributions of B_y and J_z across the x axis for both cases 1 and 2, when the tape is a normal conductor (with negligible magnetization). In neither case is there a magnetic force. In case 2, the current density is I_{b0}/wh and the field outside the tape is $B_{b0} \equiv \mu_0 I_{b0}/2w$. We may now apply superposition to determine the magnetic force when there is both current and an external magnetic field. We then find the standard result for the force along the x axis, as following from Eq. (3):

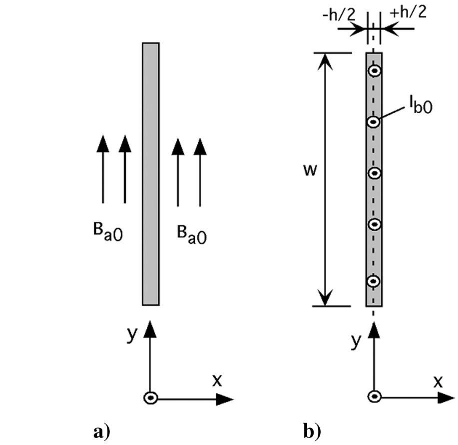

Fig. 1 Cross section of tape with either a) external field B_{a0}, or b) current I_{b0}.

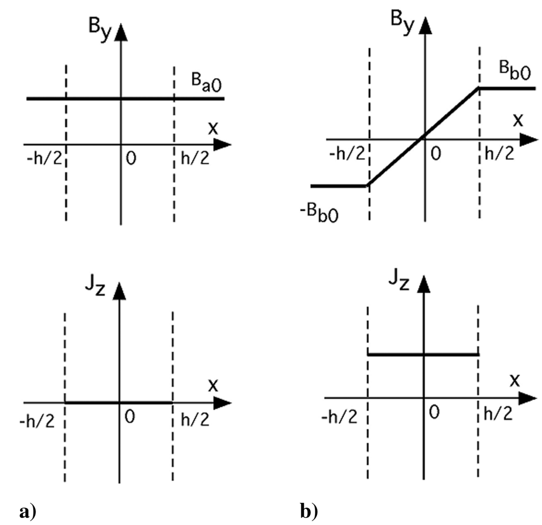

Fig. 2 Field and current density across normal conductive tape for cases a and b of Fig. 1; field as produced by I_{b0} in case b.

$$-F_x = \int \mathrm{d}V J_z B_y = \int_{-h/2}^{h/2} Lw\,\mathrm{d}x \frac{I_{b0}}{wh}\left(B_{a0} + \frac{\mu_0 I_{b0}}{2w}\frac{2x}{h}\right) = L I_{b0} B_{a0} \tag{11}$$

Figure 3 shows the distributions of B_y and J_z when the tape segment is a superconductor, where both field and current vanish except in thin layers (grossly exaggerated in the figure) on either side. Note that the signs of J_z and $\mathrm{d}B_y/\mathrm{d}x$ are equal at every point, in agreement with Eq. (10). In the London model, both B_y and J_z would vary exponentially with x but the following results are independent of the actual variation.

Note also that there is no net current in case 1, whereas the area under the J_z graph in case 2 equals I_{b0}/wh as in Fig. 2. Although there is no physical difference between currents in the two cases [8], one can still say that case 1 involves diamagnetic currents and case 2 involves (charge) transport currents.

Again, there is no magnetic force in either case. Applying superposition, we now determine the magnetic force when there is both current and an external magnetic field. Using Eq. (10), we find

$$-F_x = \int_{-h/2}^{h/2} Lw\,\mathrm{d}x \frac{1}{\mu_0}\left(\frac{\mathrm{d}B_{ya}}{\mathrm{d}x} + \frac{\mathrm{d}B_{yb}}{\mathrm{d}x}\right)(B_{ya} + B_{yb}) \tag{12a}$$

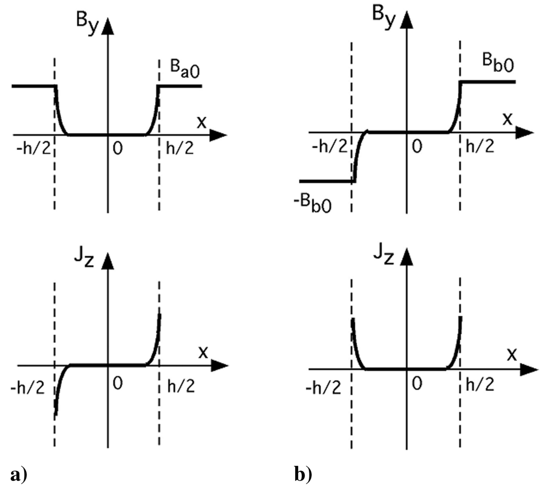

Fig. 3 Field and current density as in Fig. 2, for superconductive tape.

or

$$-F_x \times \frac{2\mu_0}{Lw} = \int_{-h/2}^{h/2} \mathrm{d}x \frac{\mathrm{d}}{\mathrm{d}x}(B_{ya} + B_{yb})^2 \Rightarrow \Delta(B_{ya} + B_{yb})^2 \tag{12b}$$

$$= [(B_{a0} + B_{b0})^2 - (B_{a0} - B_{b0})^2] = 4B_{a0}\frac{\mu_0 I_{b0}}{2w} \tag{12c}$$

where Δ represents change across the tape thickness. The result in Eq. (12c) is just as in Eq. (11).

IV. Force on a Superconductor Shielding a Current

Figure 4 shows the arrangement of a superconductive tape shield enclosing a tape segment of a loop carrying a current (whether this inner tape is a superconductor or a normal conductor); all x dimensions are small compared with w. In an actual situation, the conductor must be supported mechanically inside the shield so that they move together.

There is clearly no force on the inner conductor. The right (R) and left (L) parts of the shield will set up diamagnetic currents, due to the "outside" field $B_{b0} \equiv \mu_0 I_{b0}/2w$, in surface layers left and right, respectively. With the shield carrying no net current, diamagnetic surface layers will also set up next to the respective outer sides of parts R and L. Figure 4 shows the directions of B_y and J_z inside the surface layers of the shield, which can be readily drawn by comparing with the graphs in Fig. 3. Note the difference in directions of B_{b0} on the outer sides of the shield, that is, opposite to the case of the external field; the opposite directions are indeed responsible for the force acting on the shield, as shown next.

Following Eq. (12b), we again recover the result expressed in Eq. (11):

$$-F_x \times \frac{2\mu_0}{Lw} = \Delta_R(B_{ay} + B_{by})^2 + \Delta_L(B_{ay} + B_{by})^2$$

$$\Rightarrow (B_{a0} + B_{b0})^2 - (B_{a0} - B_{b0})^2 = 4B_{a0}\frac{\mu_0 I_{b0}}{2w} \tag{13}$$

Δ_L, Δ_R being variations across the L and R parts; contributions from the layers next to their inner sides balance each other.

The results in Eqs. (12c) and (13) [and in Eq. (11), too] can be summarized as

$$-F_x = Lw \times \left[\frac{B_y^2}{2\mu_0}(\text{right side}) - \frac{B_y^2}{2\mu_0}(\text{left side})\right] \tag{14}$$

where $B_y^2/2\mu_0$ is magnetic pressure [8]. Equation (14) just expresses the difference between magnetic-pressure forces on both (most) external sides, where the magnetic field is made of the applied field and the field generated by the current (I_{b0} here) flowing along the full loop. The force is thus independent of cross section structure.

In conclusion, a Lorentz force equal to that exerted on the loop segment if unshielded, acts on the superconducting shield.

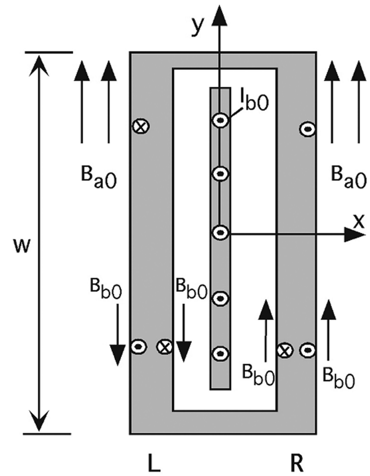

Fig. 4 Tape loop segment carrying current I_{b0} and surrounded by superconductive shield under external field B_{a0}; current directions as in Fig. 1.

V. Conclusions

It had been claimed that enclosing a segment of a wire loop within a superconductor, to magnetically shield a segment from an external uniform magnetic field, could result in a net Lorentz force on the full loop. We have here shown, however, that the force on the enclosing superconductor is the same force acting on the loop segment if left unshielded, and so no net force can be achieved. This is in agreement with Newton's third law for the mutual forces between the wire loop and the distant current distribution producing the uniform magnetic field.

As detailed in our analysis, there exist London layers of current density and accompanying magnetic field next to the inner sides of the superconductor, which shield its interior from the magnetic field that lies between the loop segment and superconductor, which is due to the current flowing in the segment. Because no net current flows in the shield, there are London layers next to the outer sides too, resulting in a magnetic field outside the entire cross section, in addition to the external magnetic field. Those London layers are subjected to volume forces that result in a net magnetic force on the shield.

Because the London layers add to no net current in the superconductor, the circulation of the magnetic field along a closed line outside the cross section, in its plane, is just due to the current flowing in the loop segment, as it follows from the ampere equation. One could then just ignore the structure of the cross section and say that, outside it, the addition of the uniform external magnetic field and the nonuniform field due to the current in the loop, do result in a nonuniform magnetic pressure, which produces the same net force whatever the cross section structure.

Acknowledgment

Work by J. R. Sanmartín was supported by the Ministry of Education and Science of Spain under Grant ESP2004-01511.

References

[1] Lorenzini, E. C., and Sanmartin, J. R., "Electrodynamic Tethers in Space," *Scientific American*, Vol. 291, No. 2, 2004, pp. 50–57.

[2] Yamagiwa, Y., Watanabe, S., Katanegi, K., and Otsu, H., "Innovative Interplanetary Transportation System Using Electrodynamic Tether and Magnetic Coil (Mag-Tether)," *25th International Symposium on Space Technology and Science*, Japan Society for Aeronautical and Space Sciences ISTS 2006-b-46, June 2006.

[3] Spenny, C. H., Dell, C. O., and Bailey, W. F., "Investigation of Forces in a Shielded Conductor," *Proceedings of 4th International Conference on Tethers in Space*, Smithsonian Institution, Washington, D.C., April 1995, pp. 1067–1076.
[4] Jackson, J. D., *Classical Electrodynamics*, 2nd ed., Wiley, New York, 1975.
[5] Bergamin, L., Izzo, D., and Pinchook, A., "Propellentless Propulsion in Magnetic Fields by Partially Shielded Current," *57th International Astronautical Congress*, International Astronautical Federation 06-C4.6.05, 2006.
[6] Lorenzini, E. C., "Novel Tether-Connected Two-Dimensional Structures for Low Earth Orbits," *Journal of the Astronautical Sciences*, Vol. 36, No. 4, 1988, pp. 389–405.
[7] Sanmartin, J. R., and Martinez-Sanchez, M., "Radiation Impedance of Orbiting Conductors," *Journal of Geophysical Research*, Vol. 100, No. A2, 1995, pp. 1677–1686. doi:10.1029/94JA02857
[8] Landau, L. D., and Lifshitz, E. M., *Electrodynamics of Continuous Media*, Pergamon, London, 1960.
[9] Rose-Innes, A. C., and Rhoderick E. H., *Introduction to Superconductivity*, 2nd ed., Pergamon, London, 1978.

R. Myers
Associate Editor